Damage Tolerance Analysis of a Helicopter Component

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ABSTRACT

Fatigue critical helicopter components are, in general, subjected to complex high frequency dynamic loading. Due to these high frequencies small flaws growing at relatively slow rates can propagate to failure in a short period of time. Consequently, the demonstration of damage tolerance for those components must include the analysis of near-threshold crack propagation, i.e. growth in the low-to-mid stress intensity factor range (ΔK) regime. This paper presents a fatigue crack growth analysis of a helicopter airframe component, which was used as part of a round-robin study into helicopter fatigue, using a non-similitude based crack growth law, termed the '*generalised Frost-Dugdale law*'. The component geometry, material data and complex loading spectrum were distributed to the helicopter structural analysts worldwide with the objective of assessing the capability of current structural analytical tools to predict fatigue crack growth and thus incorporate damage tolerance into helicopter design. The generalised Frost-Dugdale model utilised companion fatigue coupon data tested under a similar spectrum to derive the necessary two input parameters, which along with a finite element computed width correction factor, lead to good agreement between the prediction and the component's experimentally derived crack growth. The example presented in this paper demonstrates how the helicopter industry (amongst others) can accurately apply damage tolerance design principles both in the initial design as well as to ensure continued airworthiness.

Keywords: helicopter fatigue; fatigue crack growth; fatigue modelling; non-similitude.

1. INTRODUCTION

The use of linear elastic fracture mechanics (LEFM) methodology to characterise the growth of fatigue cracks in metals is well established in the design of aerospace structures, especially in the fixed-wing aircraft. However, there is very little work done on the application of fracture-mechanics to the rotorcraft. It is widely known that the task of predicting fatigue crack growth life on helicopter dynamic components is very difficult, due to their complex component configuration, characteristic of materials used and complex high frequency dynamic loading. The current design philosophy for helicopter rotary component is still based on the safe life approach, where manufacturers impose a life limit on their production parts. Safe life design is based on an assumption of linear cumulative fatigue damage (LCD), known as Miner's rule [1]. The periodic nature of helicopter component loading is assumed to cause cumulative damage. The resulting damage is related to amplitude of the applied loading as well as the number of load cycles applied to the component. The allowable damage is ultimately related to aircraft flight profiles and flight times. Safe life analysis assumes that there are no pre-existing defects in the components.

In an attempt to reduce the over-conservatism in the helicopter component life prediction, there is a growing interest in using the Damage Tolerance Design Methodology, which has been successfully implemented in airframe structures, to determine the fatigue life of helicopter dynamic components. The use of damage tolerance analysis for helicopter component design is almost non-existent prior to recent times. The damage tolerance philosophy assumes a deterministic process for fatigue fracture where a deterministic crack growth function, constant material properties and a specific initial (discontinuity) size (a preexisting flaw is assumed) are employed. This approach was found very useful and adequate for maintaining a high level of safety for operational aircraft and it can be readily applied to the design process of new aircraft structures. Lincoln [2] pointed out that the work performed on the HH-53C and HH-60A helicopter by the United State Air Force (USAF) has shown that the damage tolerance approach is viable for helicopters. In addition, the application of a damage tolerance based design philosophy to helicopter components will allow a better understanding of stress and loading these of dynamic components and allow optimisation of components that had originally a short fatigue life, as well as, shortening the life of components that had originally extremely long life to allow additional weight saving. The paper in ref. [2] contained the discussion and recommendations that are given for future damage tolerance assessment of existing helicopters and on the incorporation of damage tolerance capability in new designs. However, it can be seen that a considerable amount of work would be required before helicopter component risk assessment based on crack growth can be applied.

Fatigue critical helicopter components are, in general, subjected to complex high frequency dynamic loading. Due to these high frequencies, small flaws growing at relatively slow rates (i.e. ~10^{-10} m/cycle) can propagate to failure in a short period of time. Consequently, the design against high-cycle fatigue failure must include the analysis of near-threshold fatigue crack propagation, i.e. Region I or growth in the low-to-mid stress intensity factor range (ΔK) regime (also known as microscopic region with crack dimensions ranging from 0.01 to 1 mm). During the last decades, the efforts of using LEFM methods for analysing near-threshold crack growth has pointed out the so-called 'problem of short cracks' [3-5]. Several issues concerning short crack growth in Region I have been identified: (i) *da/dN* higher than what would be predicted by a long-crack law, such as the Paris law [6] for a given ΔK; (ii) a decrease in *da/dN* with increasing ΔK; (iii) growth at ΔK values lower than the long-crack threshold; (iv) a growth rate strongly dependent on the material microstructure. This anomalous crack growth can be due to breakdown in continuum-mechanics-based LEFM conditions. Such 'short crack' phenomenon made the analytical predictions near the threshold fatigue regime extremely difficult and it is an area of interest of many researchers.

Till now, the near threshold fatigue analysis could only be determined by empirical means, where the standard threshold data of ΔK are commonly determined for long cracks. Laws based on the governing similitude hypothesis assume that crack growth rate v ΔK data is crack-size independent. However, the validity of LEFM-based near threshold fatigue analysis in the Region I, characterised by short cracks, has been challenged by several scientists in the field [7]. Predictions in the near-threshold region, using this similitude based approaches often give highly erroneous results [8, 32]. It has been argued that the assumptions of continuum mechanics and the ΔK concept are violated because of micro-structural features. However, the transition from valid to invalid conditions does not occur abruptly. Many mechanical parameters such as the range of J integral, ΔJ and the range of strain intensity factor, ΔK_ε have been used in attempts to resolve the problems encountered in near-threshold fatigue crack growth predictions. Each of them is only valid for the relatively narrow ranges of test conditions. Since the use of ΔK to characterise the fatigue crack growth, especially in Paris region has been successfully employed in many engineering applications, a single analysis methodology based upon the use of ΔK analysis to predict the near-threshold fatigue crack growth is desired.

In recent years, the Australian Defence Science and Technology Organisation (DSTO)'s quantitative fractography (QF) examination of crack growth in Region I (for 0.01 mm $\leq a \leq$ 10 mm) could not find any evidence to support the conventional 'short crack' effect. For example, Molent *et al.* [9] highlighted a smooth, continuous and monotonically increasing crack growth in their compendium of F/A-18 aircraft fatigue crack growth data. Their work, which examined more than 300 different cracks in various F/A-18 related full-scale fatigue tests and the associated coupon test programs under spectrum loading, revealed a near log-linear relationship when plotted as logarithm of crack depth versus linear life. This relationship holds for crack growth from starting lengths of near microns up to lengths of 10 mm. This log-linear growth characteristic was first observed and reported by Frost and Dugdale [10] in their centre-notched constant amplitude fatigue loading experiments. Liu [11] subsequently confirmed the log linearity reported in [10] and in his review paper [12] he highlighted that for short cracks *da/dN* was proportional to the crack length. As such, a resurgent interest has arisen to develop a crack growth model that accounts for the smooth transition of micro- (Region I) to macro-cracking (Region II) as the small flaws or defects arising either from corrosion or manufacturing deficiencies enlarge, grow and form a large crack that can be seen by the human's naked eye. It is believed that such a transition behavior can be achieved by introducing another mechanical parameter into the crack growth rate model. As such, Barter *et al.* [13] first proposed a ΔK-based crack size-dependence crack propagation law, termed '*generalised Frost-Dugdale law*' or '*non-similitude based growth law*'.

To adequately address the above issues relating to the near threshold fatigue crack growth using the damage tolerance approach, Cansdale *et al.* [14] initiated the '*Helicopter Damage Tolerance Round-Robin*' challenge study in the '*Workshop on Fatigue Design of Helicopter*' in May 2002. This round-robin problem was distributed to the helicopter experts worldwide with the objective of assessing the current existing fracture mechanics based tools to analytically predict the fatigue crack growth in round-robin component.

2. CRACK GROWTH RATE MODELS

2.1. Fracture mechanics based crack growth laws

The study of fatigue crack growth begun in early 1950s. One of the most important contributions was the correlation of the stress intensity factor, K with the fatigue crack growth rate, *da/dN*. In 1961, Paris *et al.* [6] postulated the use of $K_{\max}$ to describe fatigue crack propagation. Subsequently in early 1963, Liu [15] published the first paper that implied that *da/dN* could be related to stress intensity factor range, ΔK, where he implied that for a wide plate, *da/dN* is linearly proportional to ΔK^2. Shortly after that, Paris and Erdogan [16] found that plots of *da/dN* against ΔK gave straight lines on log-log scales for Region II (also known as 'Paris region') in a typical crack growth rate curve for aluminium alloys. This led to the so-called 'Paris growth law', viz:

$$
\begin{aligned}
\frac{da}{dN} &= C\Delta K^{m} \quad where\ \Delta K = \beta\Delta\sigma\sqrt{\pi a} \\
\frac{da}{dN} &= C\left(\beta\Delta\sigma\sqrt{\pi a}\right)^{m} \\
&= \left[C\left(\beta\sqrt{a}\right)^{m}\right]\Delta\sigma^{m} a^{m/2}
\end{aligned}
\tag{1}
$$

Here, m and C are experimentally determined material constants and the prediction of the fatigue life is very dependent on these constants. Generally, m is taken 3 for common aluminium alloys (such as AL 7075-T6 and AL 2024-T351) and structural steels. While the constant C can be estimated using a measured crack propagation curve. Since fatigue crack propagation measurement requires precise equipment, the determination of the constant C is usually empirical and relatively complicated.

The Paris law currently provides the basis for most of the life predictions of structures made using the LEFM-based approaches and relies on the validity of the similitude hypothesis. This similitude concept indicates that at the same value of ΔK and $K_{\max}$, the crack tip stresses and strains are the same, regardless of the in-plane specimen geometry and applied load. It was later found that the crack growth rate curve is not linear for all ranges of ΔK. A plot of log *da/dN* versus ΔK gives a sigmoidal curve as shown in Figure 1. The curve may be divided into three regions, i.e. (i) Region I: initiation and threshold, (ii) Region II: stable crack growth (also known as 'Paris region') and (iii) Region III: unstable fracture. The first two regions constitute the primary portion of the fatigue life and both prove to be of primary interest from the theoretical and practical points of view. Region III crack growth life is very short compared to the total fatigue life, so it is not important from a practical point of view. Therefore, modelling of the unstable fracture region will not be discussed further.

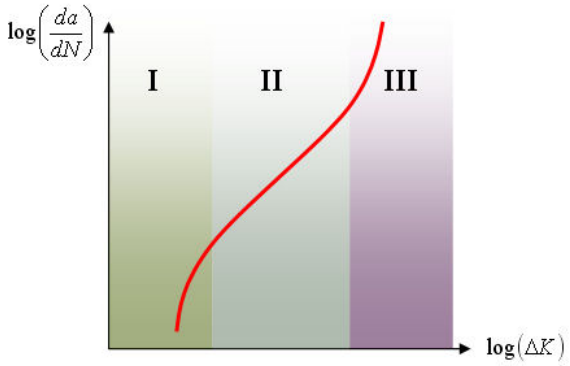

Fig. 1 Schematic illustration of typical fatigue crack growth rate data for aluminium alloys

Even though Region I has been extensively studied in recent years, a complete understanding of phenomenology, mechanisms and physical nature of the processes in this Region I is far from being achieved. Many studies related the physical features of Region I with a threshold value ΔK_{th} where the crack growth rate goes asymptotically to zero as ΔK approaches this ΔK_{th}. Here, an operational definition, based on the ΔK corresponding to a rate of 10^{-10} m/cycle, has been established [17]. A physical explanation of the fatigue threshold is that if nominal load ranges are very small, then local cyclic strains around the stress concentrator do not exceed the elastic range. This means that no plastic deformation (dislocation movement) occurs around the grain scale stress concentrators, micro-cracks do not nucleate and local damaging of material does not develop. As such, ΔK_{th} serves as an idealised model of the physical threshold from the viewpoint of LEFM.

The crack growth relations in the threshold region have been proposed by Donahue *et al.* [18] as:

$$\frac{da}{dN} = \hat{C}\left(\Delta K - \Delta K_{th}\right)^{m} \tag{2}$$

where $\hat{C}$ is constant. Eq. (2) is one of the most popular modifications to the Paris law to account for the fatigue threshold. The value of ΔK_{th} in Eq. (2) is very difficult to measure experimentally as it can be very different in value and in nature. Such inconsistencies can lead to non-conservative design and reliability assessments. Nevertheless, a number of theoretical models have been proposed to analytical determine the ΔK_{th} value. One of them is the well-known El Haddad model [19] which describes the influence of the crack length a on ΔK_{th}. The El Haddad model has more than two experimentally determined constants, which can vary depending on micro-structural features, material properties, loading condition, environment, etc. These circumstances promoted the development of an alternative engineering approach to the formulation of improved methods to predict fatigue lives of structure by combining the crack growth in Regions I and II. One of the promising mechanics approaches to describe the growth in the near-threshold regime involves the concept of *non-similitude*. This concept suggests that the increment in the crack length per cycle (da/dN) is not solely governed by ΔK, but is also dependent on another mechanical parameters, such as crack size, a, or nominal stress, σ_n.

2.2. Non-similitude based crack growth modelling

The Paris equation was not the first law describing crack growth. The first law (according to Frost *et al.* [20]) can be attributed to an early Australian DSTO researcher, Head [21]. Subsequently, Frost and Dugdale [10], using Head's observation of self-similar crack growth, expanded Head's law and reported that crack growth for centre cracked panels tested under constant amplitude loading could be described via a simple log-linear relationship, viz:

$$Ln(a) = \beta N + Ln(a_0) \quad or \quad a = a_0 e^{\beta N} \tag{3}$$

where N is the 'fatigue life', β is a parameter that is geometry, material and load dependent, a is the crack depth at time N and a_0 is the initial crack-like flaw size. This linear relationship between $ln(a)$ and N has also been reported, for physically short cracks, by a large number of researchers including Harkegard, Denk and Stark [22], Nisitani *et al.* [23], Kawagoishi *et al.* [24], Caton *et al.* [25], Murakamia and Miller [26], Polak and Zezulka [27] and Tomkins [28]. Indeed, Polak and Zezulka [27] found that this approximation held for sub-micron cracks.

Eq. (3) can be rewritten in terms of da/dN as:

$$\frac{da}{dN} = \widetilde{C} \Delta\sigma^n a \tag{4}$$

where $\widetilde{C}$ is empirical constant derived from experimental data and the stress exponent, n was found to be 3 from the Frost and Dugdale's [10] experimental results for various metals and alloys under constant amplitude loading. Hence, Eq. (4) is sometimes also referred to as the 'stress-cubed' law due to its cubic stress dependency. Here, Eq. (4) represents a crack size dependence law where it was suggested that the propagation rate of a fatigue crack should be proportional to the current size of the crack. This differs from the similitude based laws that are based on Eq. (1).

It has been shown more recently that Eq. (4) can be effective in correlating small fatigue crack growth. For example, Nisitani *et al.* [23] showed that Eq. (4) successfully correlates small crack growth in 0.45% C steel and an Fe -3% Si alloy, where the cracks monitored were ~0.01 to 2 mm in length.

However, Eq. (4) has limited application to small cracks lengths only (when compared to the net-section size). It may not be applicable if applied to large crack lengths (when compared to the net-section size), different crack geometries, different component geometries, or; component loading/boundary conditions, due to the absence of a geometry and loading/boundary condition factor in the model.

To overcome these limitations, Barter *et al.* [13] first proposed a modified version of ΔK-based Frost-Dugdale model as given in Eq. (5). In ref. [13], they presented a range of examples that support the Frost and Dugdale's hypothesis that (i) as a first estimation, the $ln(a)$ versus N relationship can be taken to be linear and that this is true both for constant amplitude and spectrum loading. Additionally, they have shown that (ii) for a given spectrum the variation in crack growth rate is approximately dependent upon the stress cubed, which means the stress exponent should be 3. Both remarks (i) and (ii) from their numerous experimental observations have suggested a possible generic growth law, termed the '*generalised Frost-Dugdale law*', of the form [13]:

$$\begin{aligned} \frac{da}{dN} &= \overline{C} a^{1-\gamma/2} \Delta K^{\gamma} \\ &= \overline{C} \left(\beta \Delta\sigma \sqrt{\pi a}\right)^{\gamma} a^{1-\gamma/2} \\ &= \left[\overline{C} \left(\beta \sqrt{\pi}\right)^{\gamma}\right] \Delta\sigma^{\gamma} a \end{aligned} \tag{5}$$

where $\overline{C}$ is a material constant, γ is approximately 3, and ΔK is the crack driving force. Spagnoli [29] has also proposed a similar law as Eq. (5) where she suggested that the crack surface can be modeled as self-similar invasive fractals. Eq. (5) implies that the fatigue crack growth in Region I and II essentially does not conform to the similitude concept, because the crack growth rate of the crack is based on the applied values of both ΔK and crack size. This law is formally contradictory to the Paris similitude based growth law. As such, conventional continuum LEFM rules do not apply to the Eq. (5). Indeed, in the slow growth regime crack growth in rail steels [30] and several aerospace alloys [13, 30, 31], can be well described by the generalised Frost-Dugdale law.

3. DESCRIPTIONS OF THE HELICOPTER ROUND-ROBIN PROBLEM

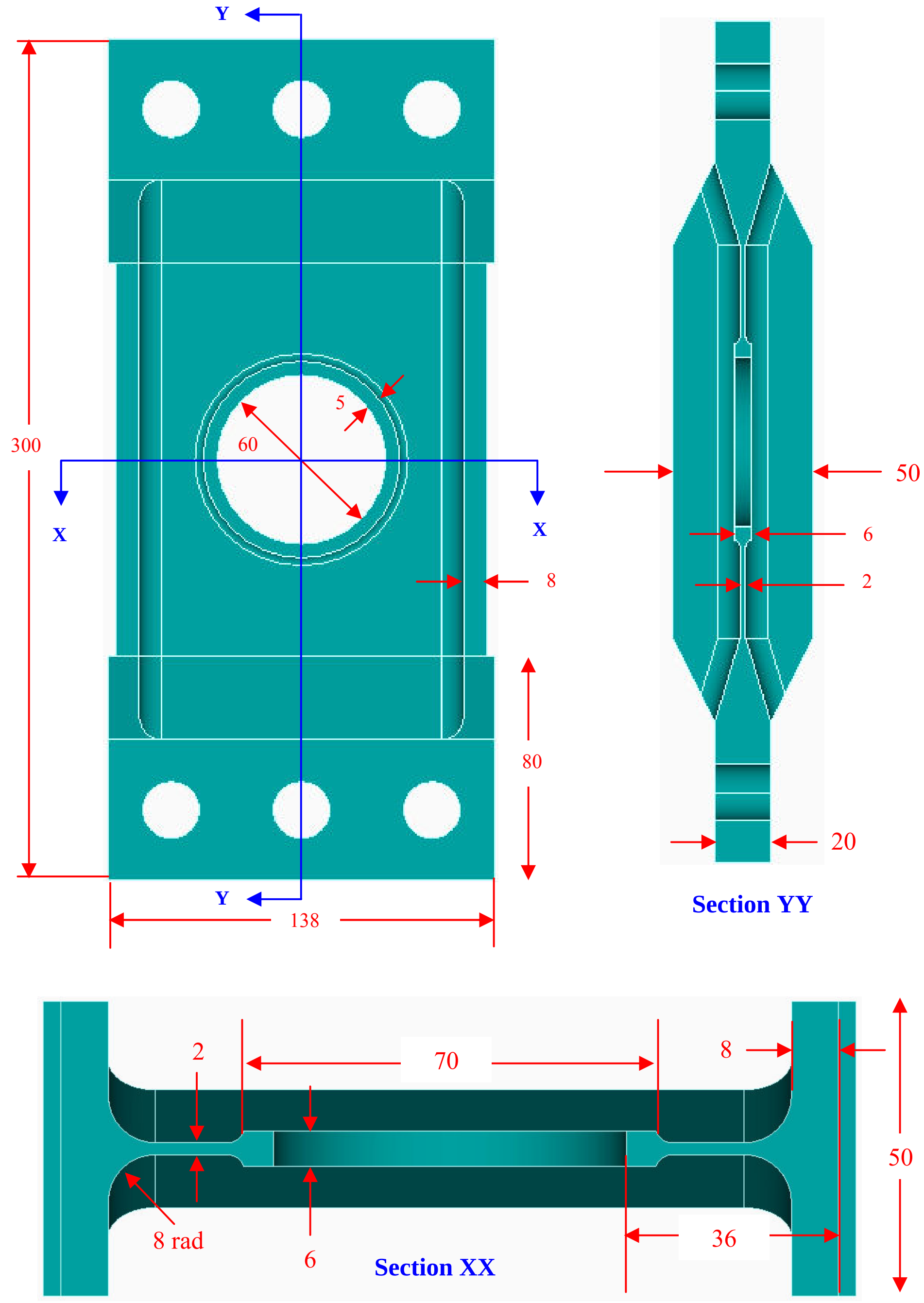

Fig. 2 Schematic diagram of round-robin component (all dimensions are in mm)

The '*Helicopter Damage Tolerance Round-Robin*' challenge problem was recently proposed by Cansdale and Perret [14] and Irving, Lin and Bristow [32]. The aim of this round-robin study was to compare the analytical prediction made using the damage tolerance approach with those experimental results on the same material in the same component geometry and subjected to the same spectrum. The round-robin helicopter component was a flanged plate with a central lightning hole made of the 7010 alloy, see Figure 2. It was subjected to a representative helicopter spectrum loading, called Asterix. The round-robin experimental results were not provided to all the participants, but all the essential crack growth parameters were given, such as initial crack length, 7010 alloy crack growth rate data and the surface values of stress intensity factor solutions. A range of predictions were submitted to the round-robin organiser using a total of 8 different software packages [32]. None of the 27 predictions received matched the test data, especially in the early stage of the crack growth where the predicted initial growth from the 2 mm defect was very slow. Newman *et al.* [33] subsequently used the 'average' stress intensity factors obtained from a three-dimensional BEASY analysis in his prediction. Newman *et al.*'s analytical predictions were made using FASTRAN-II, which is based on the 'plasticity-induced' crack closure concept [34], and they agreed very well in the early stages of crack growth. But the model predicted faster crack growth through the thin (2 mm) section. The predicted crack growth life at a crack length of 25 mm was 30% short of two test results conducted under the Asterix helicopter spectrum.

In this present paper, attempts were made to analytically predict the round-robin complex crack growth using the concept of non-similitude where it was proposed that the fatigue crack growth is essentially crack-size dependent, as discussed earlier.

3.1. Crack configuration

The study required the participants to predict crack growth from a 2 mm (radius) corner defect on the inner edge of the large central hole to the failure, where failure was defined as a projected surface crack length of 25 mm. A schematic diagram of the crack growth evolution is given in the Figure 3. As can be seen in Figure 3, the round-robin problem involved rather complex crack configurations during the transition from 6 mm reinforced hole-edge to a 2 mm thick web, i.e. from a corner crack (3D crack) to a normal-through crack (2D crack). It should be noted that the crack was seen to grow for a considerable part of its life as a part through crack. The crack length, *c*, is measured along the surface.

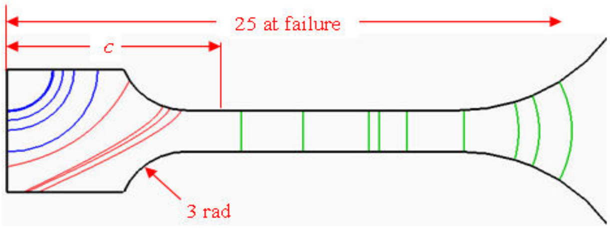

Fig. 3 Schematic diagram of the round-robin crack growth evolution (all dimensions are in mm)

3.2. Material

The round-robin component was made of AL 7010-T73651 (also known as AL 7010-T7451). It is a high strength/high toughness and stress corrosion cracking resistant aluminium alloy. Tensile test results indicate the average room temperature tensile strength was 502 MPa and the yield strength was 440 MPa for longitudinal grain orientation test specimens. For the same test conditions, the tensile elongation (ductility) was given as 13%. The average toughness at 22°C for a (L-T) orientation test specimen was 33.4 MPa√m. As in [14, 32], the round-robin coupon was assumed to have a Young's modulus of 70,000 MPa and a Poisson's ratio of 0.3.

3.3. Helicopter load spectra

The Asterix load sequence used in this study was derived from strain data measured on a helicopter lift frame. The maximum and minimum normalised stress points in the spectrum were given as 100 and -4 respectively. The normal distribution for the stress ratios (as given in [35]) indicates that the stress ratio of 0.8 dominates the load spectrum, which indeed is an extremely high stress ratio spectrum with some negative stress ratio excursions as shown in Figure 4. The Asterix spectrum represented 190.5 flights or 140 sorties and composed of 371,610 cycles. The largest far field stress in the spectrum was set to 130 MPa.

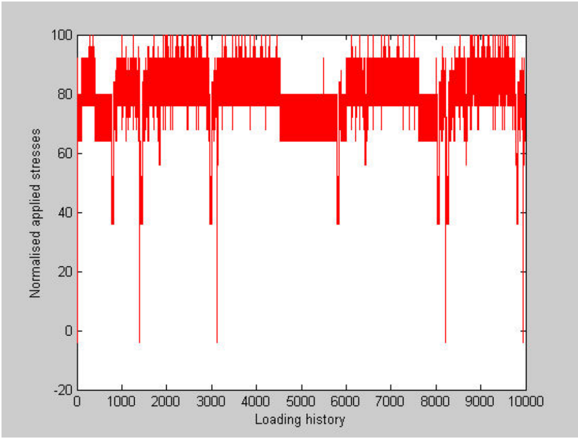

Fig. 4 Asterix spectrum loading

4. STRESS INTENSITY FACTOR ANALYSIS

In the '*Helicopter Damage Tolerance Round-Robin'* exercise, participants were supplied with a stress intensity boundary correction factor, *F* (for the crack at the flange surface of the round-robin component from 2 to 35 mm long), derived from three-dimensional finite-element analysis, as shown in Figure 6 (a dark blue continuous curve). The majority of the predictions under-predicted the initial phase of crack growth (i.e. crack propagation in the central 6 mm thick flange) using the provided stress intensity factor solutions. Hence, the validity of the provided compliance *F* function to represent the test configurations has been questioned by several researchers [33, 35]. Using the limiting values of the stress concentration factor based on the net-section stress calculated by Tada *et al.*[36] and Newman and Raju [37], Newman [33] concluded that the stress intensity factor solution from BEASY is most likely the correct solution for the round-robin for the initial crack growth phase. Newman's BEASY *F* solution is shown in Figure 6 (a pink continuous curve). It was noticed that there is a significant difference in the 2 to 5 mm crack size range between the sponsor's round-robin and Newman's BEASY *F* solutions.

Initial crack growth is undoubtedly an importance crack propagation phase in fatigue life estimation from an engineering point of view, because it will significantly influence the end results. Due to the difference, authors conducted a separate independence finite element analysis (FEA) to calculate the 3D corner crack (2 to 5 mm) stress intensity factor solutions, using a new computational program, called FAST (**FA**ilure **ST**ructure Analysis), developed by several Australian Monash University researchers. The FAST computer code uses the Vijayakumar and Atluri's 3D stress field formulation to estimate the 3D stress intensity factor solution. It should be noted that the polynomial formulation used by Vijayakumar *et al.* [38] to define the stress field is not central to this study, so it will not be further discussed in this paper, for more details see [38].

Symmetry considerations mean that it was only necessary to model half of the structure, as shown in Figure 5. The model was hex-meshed and consisted of 268,478 nodes or 61,962 9-noded CHEXA elements. It was subjected to a peak (remote) load which gave 130 MPa at the net section of the symmetrical plane. The model was then imported into the NEi/NASTRAN to compute the resultant stress field. The computational analysis took 1351.7 seconds (equivalent to 23 minutes) CPU time to complete under a Linux platform.

As can be seen from Figure 3, the round-robin fatigue crack initiated as a corner crack, grew across the thickness and then subsequently transformed into a through crack. It involved a rather complex crack evolution, especially at the transition phase (between the corner 3D crack and normal-through 2D crack). However, cracks usually have a tendency to quickly become normal-through-thickness cracks once they reach the new free surface. Therefore, for the purpose of simplicity and conservativeness in the crack growth life analysis, we used the stress intensity factor solutions computed from the FAST program to predict 3D corner crack growth at 6 mm thick section (i.e. growth from 2 to 5 mm). While in the thinner section (i.e. growth from 5 to 25 mm), we used the original round-robin stress intensity factor solution (provided by the round-robin sponsor). This has resulted in a tuned round-robin *F* solution as shown in Figure 6 as a red continuous curve. A significantly large variation of *F* solutions between the sponsor's, BEASY's and FAST's solutions was observed between crack lengths of 2 to 3 mm. As the crack propagated, the difference between the FAST's and sponsor's *F* solutions gradually diminishes and eventually merged at *a* = 4 mm onward.

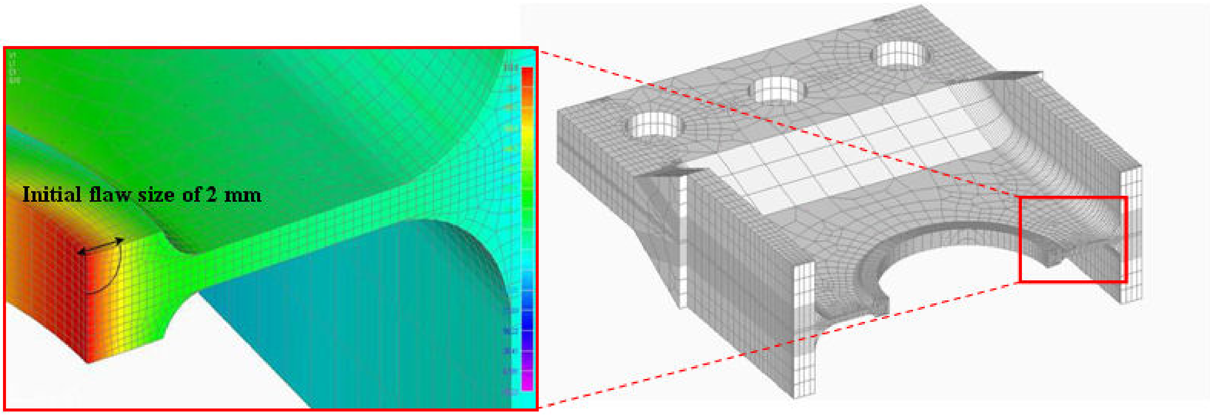

Fig. 5 Nominal stress distribution of round-robin crack configuration at the crack face

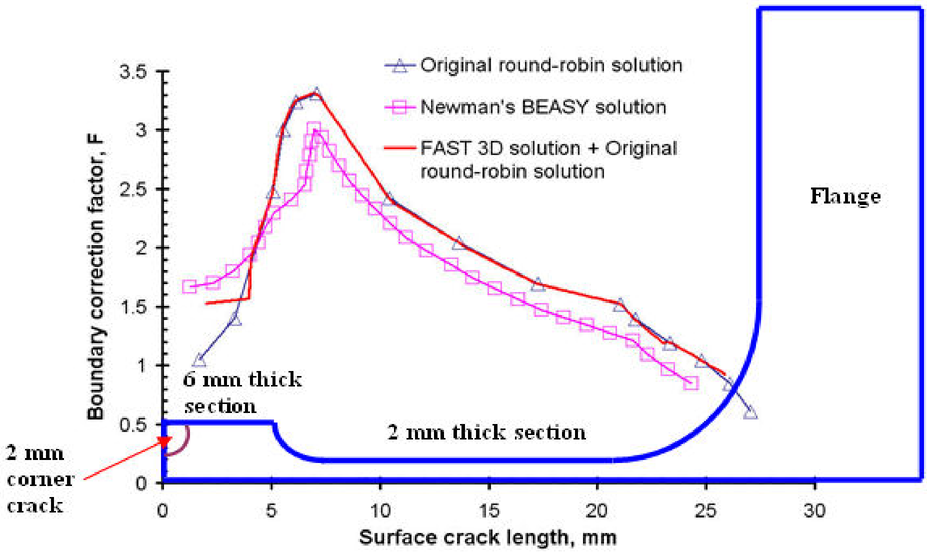

Fig. 6 *F* solutions for the round-robin crack configuration as a function of surface crack length

5. FATIGUE CRACK GROWTH LIFE PREDICTIONS

Cansdale *et al.* [14] and Irving *et al.* [32] presented the crack length histories for two test specimens. The starting defect in specimen number 1 was a spark machined quarter circular corner crack of 2 mm radius located at the hole edge. In test 2, the defect radius was 1.5 mm and the defect was grown as a fatigue crack under the Asterix spectrum up to 2 mm radius and crack growth data were recorded from this point up until failure. From Figure 7 it can be seen that there was little difference in the crack growth histories. Both start at a moderate growth rate which steadily increases to a fairly rapid rate and was then followed by a much slower rate of crack growth. Those test data were used to validate the predicted crack growth lives.

Figure 7 shows selected predictions submitted by several participants of the round-robin study. It is seen that not a single similitude based prediction followed the same qualitative crack growth behavior as observed in the test data, where most of them exhibited slower crack growth rate at the early stage of crack growth and significantly faster growth rate in the 2 mm thick section than the test data. However, the majority of the predictions at the failure (which was defined as at 25 mm crack length) are close to the test data. Newman *et al.*'s round-robin prediction made with the BEASY average stress intensity factor solutions gave good agreement with the test data, typically at the early stages of the crack growth, see Figure 8. However, the crack growths at the thinner section (2 mm thick) were too fast. The final flight hours at the 25 mm crack length was 30% shorter than the given test data. Please bear in mind that the previous predictions including the one made by Newman *et al.* [33] were all based upon the concept of similitude, i.e. in a similar form of Eq. (1).

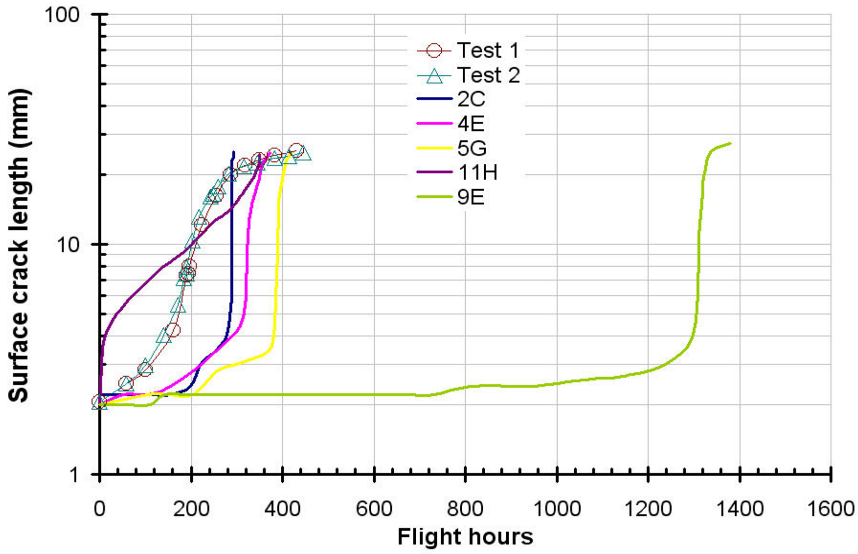

Fig. 7 Several predictions submitted to the round-robin organiser [32]

The present paper is concerned with the application of the generalised Frost-Dugdale law, as given in Eq. (5), to study fatigue crack growth under a typical helicopter load histories in AL 7010-T73651. At this point it should be noted that in [39] ΔK was defined as $\beta(\sigma_{\max}-\sigma_O)\sqrt{\pi a}$, where σ_o is the crack-opening stress, and it was necessary to adjust the constraint factor, α as the crack grew. In the present study where ΔK was kept as $\beta(\sigma_{\max}-\sigma_{\min})\sqrt{\pi a}$. Furthermore, in this study there were only two material constants, i.e. γ and $\overline{C}$ in Eq. (5), and their values were kept constant throughout the entire load history.

It is known that the Paris and generalised Frost-Dugdale models are of a different form; thus the crack growth parameters are not interchangeable [40]. The constant C and m of the Paris model can be estimated using the conventional similitude-based LEFM approach (i.e. da/dN versus ΔK curve), which rely on constant amplitude (CA) fatigue data. However, such a method can not be directly applied to the generalised Frost-Dugdale model. Further studies and more tests are needed to establish a new method to experimentally estimate the value $\overline{C}$ of the non-similitude based generalised Frost-Dugdale equation from CA fatigue data.

One of the possible approaches is plotting CA fatigue data by log-log $da/dN \times \sqrt{a}$ versus ΔK^3, where $\overline{C}$ can be obtained from the y-intercept of the curve. The authors are currently investigating this approach for crack growth in AL 7075-T6 and AL 2024-T351 aluminium alloys where CA fatigue data of these two materials are widely available in the online literature. Unfortunately, this $da/dN \times \sqrt{a}$ versus ΔK^3 approach has not been studied in AL 7010-T73651 material because the crack length parameter (i.e. a) of the AL 7010-T73651 CA fatigue data were not being provided by the round-robin organiser (only da/dN versus

ΔK fatigue data were available). Indeed, the crack length parameter plays an important role in the non-similitude based crack growth prediction, where the fatigue growth rate was suggested to be crack size-dependent.

For the purpose of simplicity in the present analysis, the value of $\overline{C}$ was determined by using a simple curve fitting technique, where $\overline{C}$ was manually adjusted to match the 2-to-4 mm corner crack round-robin test data. This approach yielded a value of $\overline{C} = 1.28 \times 10^{-11}$. This value of $\overline{C}$ value was then used to predict the rest of the fatigue crack growth, viz the growth from 4 to 25 mm. Whilst γ was taken to be 3 as a cubic stress dependency was assumed. The resultant predictions are given in Figure 8.

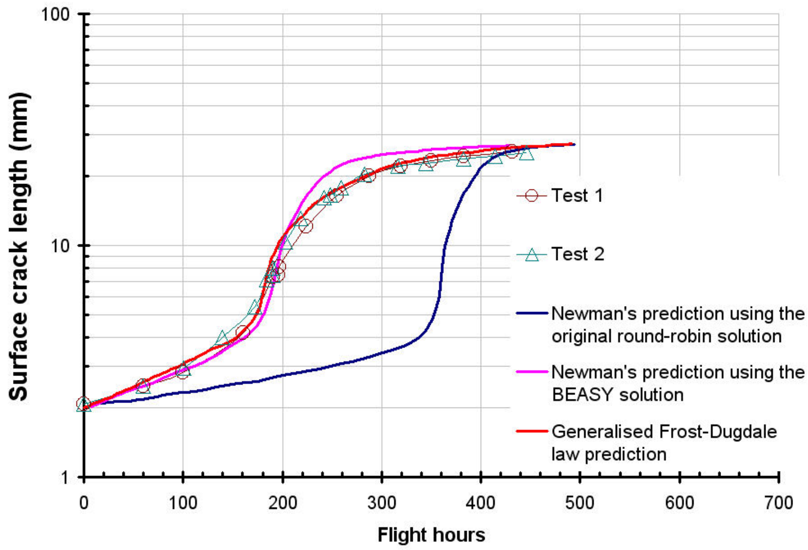

Fig. 8 Comparison of the round-robin test data and predictions

6. CONCLUSIONS

As shown in Figure 8, the generalized Frost-Dugdale law produced a crack length history curve that matched the round-robin test data extremely well.

Newman *et al.* [33] pointed out that:

> '*The early stages of crack growth compared very well with the test data. But, again, the crack growth rates in the 2 mm thick section were too fast*'

This shortcoming is largely removed in the present study, where the crack growth in the 2 mm thick material section matched the average of the two tests exceptionally well. Moreover, the predicted life at failure (i.e. at $a = 25\ mm$) was only 15% shorter than the average test results.

It should also be noted that interest in this problem was kindled due to the conclusions reached by US Army researchers Vaughan and Chang [35], who used AFGROW and NASGRO, who stated:

(i) '*It is evident that the crack growth was under-predicted for the crack length below 5 mm and is over-predicted when crack length is more than 10 mm.*'

(ii) '*We believe more research; especially in the area of modeling crack growth near the crack growth threshold and determining experimentally the crack growth threshold values are needed.*'

The present crack growth law was specifically formulated to point (ii) [30, 41] and its ability to predict crack growth history for this complex and challenging problem is particularly pleasing. Although the generalised Frost-Dugdale law has been shown to perform very well in the present helicopter round-robin fatigue crack growth prediction, a reliable method to experimentally estimate $\overline{C}$ from the material CA crack-growth-rate data is yet to be established.

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